Report of the IAG Study Group 2.2 on
Forward Gravity Field Modelling Using Global Databases

http://www.cage.curtin.edu.au/~kuhnm/IAG_SG2.2/index.html

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Period Covered: 2003-2007

Important Note: This report has been prepared by the chair and vice-chair and is mostly based on information and feed-back obtained from members of the study group. Therefore, likely it will not cover the whole range of activities of all members, for this we would like to apologize. However, we hope that the most important aspects of the study groups activities and achievements are reflected by this report.

1. Introduction

This document presents the status report of the work undertaken by the IAG Study Group 2.2 since its creation in August 2003 after the IAG Executive Meeting during the XXIII General Assembly of the IUGG at Sapporo. The Study Group (SG) can be partly seen as a continuation of IAG Special Study Group 3.177 with special focus on forward modelling. During the period 2003 – 2007 the Study Group established its terms of references, organized its membership structure, created an internet website, held three official meetings, organized one conference session and its members were present at a number of different conferences and workshops. Apart from study group related activities it is acknowledged that there are noticeable activities accomplished by individuals outside of this study group showing the importance of the topic (see references not related to members of the study group).

A vast number of data describing the Earth's shape and internal structure (e.g. elevation, density and other geophysical parameters) are currently available. Several of these data are given globally with a continuously increasing resolution. For example the latest release of SRTM-derived terrain models (e.g. Farr et al. 2007) provide an almost continuous view of the global topography with the rather high resolution of 3-arc-sec by 3-arc-sec. Apart from many different Digital Elevation Models (DEMs) available there also exist global geological and geophysical data sets describing the Earth's interior with the 2-deg by 2-deg global crustal database CRUST 2.0 (Mooney et al. 1998) as a prominent model. The increasing number of these data allows the use of forward gravity modelling techniques (direct application of Newton's integral) in order to perform gravity field recovery and interpretation. Forward modelling results were and still are of great importance in geodetic as well as geophysical gravity field modelling (e.g. Tsoulis and Kuhn 2006). Furthermore, the comparison of forward gravity modelling results with existing gravity field models obtained from observations such as models obtained from the geodetic satellite missions CHAMP, GRACE and in future GOCE can reveal useful information on the dynamics of the Earth's interior as well as the validity of forward gravity modelling techniques.

2. Primary Objectives of the Study Group

The primary objective of this SG has been to investigate the use of different global datasets describing the Earth's shape and internal mass distribution for gravity field recovery and interpretation, using forward gravity modelling techniques (direct application of Newton's integral). Special focus of the SG was on the employment of high-resolution digital elevation models as well as datasets on the structure of crust and mantle to recover high-frequency information of the Earth's gravity field and to investigate whether these data can be used to study the behaviour of gravity inside the (topographic) masses. Furthermore, the SG tried to scrutinize existing and alternative approaches used in forward gravity modelling. Summarising, the SG's main generic objectives were as follows:

- Construction of forward gravity field models using geophysical data
- Interpretation of forward gravity field modelling results
- Application of forward modelling results in gravity field determination

3. Membership Structure

It is acknowledged that the SG's members have a wide ranging background from geodesy over geophysics to mathematics. Officially, the members have been divided into full or active members and corresponding members. However, no distinction has been done here as it is acknowledged that the same degree of activity has been demonstrated from both types of members.

Chair: Michael Kuhn (Australia)
Vice-chair: Dimitrios Tsoulis (Greece)

Full Members:
Hussein Abd-Elmotaal (Egypt)
Giampietro Allasia (Italy)
Heiner Denker (Germany)
Pavel Nóvak (Czech Republic)
Spiros Pagiatakis (Canada)
Nikolaos Pavlis (USA)
Gabor Papp (Hungary)
Dan Roman (USA)
Kurt Seitz (Germany)
Gyula Tóth (Hungary)
Yan Wang (USA)

Corresponding Members:
Irek Baran (Australia)
Miroslav Bielik (Slovak Republic)
William Featherstone (Australia)
Jakob Flury (Germany)
Simon Holmes (USA)
Michael Kern (Austria)
Jon Kirby (Australia)
Roland Pail (Austria)
Gabriel Strykowski (Denmark)
Tony Watts (UK)
K. Insa Wolf (Germany)

4. Activities of the Study Group

The material presented here has been mostly compiled from information and feed-back obtained from members of the study group throughout the period covered. In addition it also contains information of the most important activities of the SG such as the organization of session 8 on "Terrain data and geopotential forward modeling", held in 2006 at the 1st International Symposium of the International Gravity Field Service (IGFS), in Istanbul, Turkey.

4.1 Forward Gravity Modelling – Theory and Practical Application

Forward gravity modelling was and still is a central part of most techniques that model the Earth's gravity filed. In the past few years an increasing interest in forward gravity modelling could be recognized, which is partly due to the availability of results from the new geodetic satellite gravity missions (Champ, GRACE and in future GOCE) as well as the increasing availability of topography, density and other geophysical data describing the Earth's interior with ever increasing resolution. In Geodesy and geophysics, great part of forward modelling techniques are dedicated to the evaluation of terrain corrections or reductions (e.g. Takin and Talwani 1966, Zhou et al. 1990, LaFehr 1991, Parker 1995, 1996, Li and Chouteau 1998, Nowell 1999, Chakravarthi 2002,)

The evaluation of gravitational potential and/or attraction using methods different to the classical application of some sort of numerical integration has been investigated. In this regards analytical integration formulae can be regarded as an alternative to numerical integration techniques. These techniques usually have the advantage that the computational effort is much decreased with respect to classical numerical integration techniques. An analytical approach to approximate potential integrals has been presented by Allasia (2002) that can be applied to Newton's integral. Furthermore, Allasia (2004), Allasia and Besenghi (2004) and De Rossi (2004) developed and applied different algorithms for the approximation of surface data. Furthermore, analytical solutions of Newton's integral in terms of polar spherical coordinates have been presented by Tenzer et al. (2007).

A new space domain method for forward gravity modelling has been outlined by Strykowski (2003, 2006). Compared to classical forward modelling techniques such as the application of rectangular prisms the new method has considerable computational advantages as complicated mass density models of almost arbitrarily structure can be used without increased computational effect with respect to more simplified structures. This is achieved by a pre-computation and storage of the given mass density distribution, thus it is not necessary to calculate the source attraction for each field point separately. The mathematical formulation is based on power series expansions of the reciprocal distance function. The paper outlines the mathematical structure of the method and demonstrates that it can be used for gravity forward modelling. However, the method still lacks of an intelligent strategy to store the integrated mass density information and the brute-force implementation will not work because of the dimensionality of the problem is too big.

Different collaborative research between members of the SG took place over the last four years that explored aspects of forward gravity modelling techniques (Kuhn and Featherstone 2003a, Kuhn and Seitz 2005) and the application of regional and global data sets for gravity field recovery (Kuhn and Featherstone 2005, Tsoulis and Kuhn 2006). The optimal spatial resolution of crustal mass distributions for forward modelling has been discussed by Kuhn and Featherstone (2003a) while Kuhn and Seitz (2005) compare the solution of Newton's integral in the space and frequency domains. Wild-Pfeifer and Heck (2006) compared different methods for modelling mass density effects in the space and frequency domains in view of modelling topographic and isostatic effects in satellite gravity gradiometry observations. Both Kuhn and Seitz (2005) and Wild-Pfeifer and Heck (2006) use tesseroids in the space domain and a spherical harmonic representation of Newton's integral in the frequency domain. Tsoulis and Kuhn (2006) provide a review on recent developments in forward gravity modelling and its application for synthetic Earth gravity modelling and gravity recovery.

A review on the global evaluation of Newton integral in (Gauss) geodetic coordinates is given by Vajda et al. (2004). Using an exact formulation as well as spherical approximation various topographical corrections are addressed in terms of their definition, upper and lower topographic boundary and density used. Numerical aspects of the evaluation of the Newton integrals, such as the weak singularity treatment, split-up into spherical shell and terrain terms, and a requirement to integrate over the whole globe are adressed as well. Special attention is given to the so-called ellipsoidal topography of constant density. Furthermore, the ellipsoidal representation of the topographical potential and its vertical gradient has been studied by Novák and Grafarend (2005).

4.2. Gravity field modelling

Efforts have been put on the investigation of different modelling techniques for precise gravity field modelling (e.g. Kuhn 2003, Tsoulis 2003, Tsoulis and Tziavos 2003, Tsoulis et al. 2003). The application of polyhedron volume elements (PVEs) has been proven to provide a more precise geometrical description of the topographic surface than the description by rectangular parallelepiped (prism) models. The use of density models based on PVEs provides smoother and more realistic field structure for the second derivatives of the disturbing potential than the one provided by a prism model. An effort was made to compute synthetic vertical gravity gradients from a polyhedron model of the topography in a local area of Hungary (Benedek 2004). The resolution of the model used was 10 m x 10 m in a 20 km distance around the computation points. The results of simulation were compared to the data obtained from the prism model of the topography and the in situ VG measurements. Apart from a shift the field structure generated by the polyhedrons fit to the VG measurements sufficiently (±136 E in a range between 3800 E and 3200 E).

If the calculation level is close to the surface of topography, then the accuracy of gravity related quantities (e.g. gravity anomaly, gravity disturbance) can be improved significantly by a detailed description (e.g. by polyhedrons) of this surface in the vicinity of the calculation point. The improvement is in the range of a few cm in terms of geoid undulation and it shows a clear correlation with the height and its variability (e.g. Tsoulis 2003).

Gyula Tóth provided FORTRAN90 software routines for the computation of the gravitational effect of a polyhedral mesh. The software as well as a Power Point presentation can be downloaded form the SG's webpage (see section 4.7 below).

Rózsa and Tóth (2007) determine the effect of topographic masses on the second derivatives of gravity potential using rectangular prism and tesseroid bodies. Numerical results based on the global DEM ETOPO5 show that the effects can reach significant levels of 10 Eötvös in all gravity gradients at the altitude level of LEOs. Furthermore, Tóth and Völgyesi (2007) apply surface gravity gradient measurements in local gravity filed modelling over Hungary.

High frequency terrain effects over Germany based on a high resolution DEM (1-arc-sec by 1-arc-sec) have been studies in view of removing those effects from future GOCE data (Voigt and Denker 2006). The high frequency effects have been derived for gravity anomalies, deflections of the vertical and geoid heights. The major aim of the study was the derivation of optimal resolutions of terrain data for given accuracy levels. Another study used high resolution gravity and elevation data in order to look at their effect on local gravity parameters (Novák 2006)

A probabilistic inversion method was developed and tested in a mining area of Western Australia (Strykowski et al. 2005). It is based on the iterative statistical analysis of the misfit between the observed gravity field and the superimposed response of a number of elementary prismatic sources (rectangular parallelepipeds). The a priori model should rely on some geological information giving the approximate 3D extension of the source. The method is applicable if the gravitational effect of the source body to be determined can be isolated from the regional gravity signal and from the signals of other local sources. For this purpose a so-called multi-scale edge technique is used.

Looking forward to accessing the on board gradiometer data of GOCE the possibility of their geophysical inversion was investigated in the Alps-Pannonian-Carpathians region (Benedek and Papp, 2006). Here a detailed 3D model of the lithosphere is available and its improvement is desired. The spectral and space domain investigations show that a reliable estimation (± 25-$50\ \mathrm{kg/m^3}$) of the density contrast on the Moho discontinuity will be possible even if planar (prism) approximation is used for the inversion. For this purpose the gravitational effect (the second derivatives of the disturbing potential T) of both the topographic masses and the well explored Neogene-Quaternary sediments have to be removed from the satellite observations. The amplitude of the gravitational signal generated by the Moho and the surface topography may reach 1 Eötvös unit at satellite altitude. The effect of the sedimentary complex is less by about one order of magnitude. The planar approximation however, may introduce about 10% systematic distortion of the simulated gravitational gradients in the region investigated. Therefore the forward computations have to be performed in a global Cartesian coordinate system utilizing the polyhedron approximation.

Some studies on the effect of topographical masses on airborne gravimetry (Novák et al. 2003) and space borne gravimetric and gradiometric data (Novák and Grafarend 2006) have been performed. The latter study also includes effects form atmospheric masses. Gravity reductions using a general method of Helmert’s condensation method have been studied by (Novák 2007). Tenzer et al. (2003) studied the far-zone contribution to topographic effects used in athe Stokes-Helmert geoid determination method.

Some SG members were also active in gravity field modelling using different gravity data and geoid determination techniques, which is loosely associated to the aims of the group. Gruber (2003) looked into global gravity field modelling techniques and Gitlein et al. (2005) studied local geoid determination by the spectral combination technique. A comparison between Stokes integration and the application of wavelet techniques for regional geoid modelling has been performed by Roland and Denker (2005b). Kern (2004) made a comparison of data weighting methods for the combination of satellite and local gravity data while Roman et al. (2004) assessed the new U.S. National Geoid Height Model GEOID03.

4.3 Crustal and lithospheric modelling and interpretation

Studies have been dedicated towards the determination of the lithosphere's elastic thickness and its anisotropic variations (Swain and Kirby 2003a, 2003b). This has largely been achieved through development of a new wavelet-based analysis and inversion method (Kirby and Swain, 2004). A wavelet-coherence between Bouguer anomaly and topography/bathymetry data is formed, using rotated 2D Morlet wavelets arranged in a 'fan'-geometry. Both isotropic and anisotropic wavelet coefficients can be derived. These are then inverted against the predictions of a thin elastic plate model, for elastic thickness in the isotropic case, and weak rigidity direction and percentage anisotropy in the anisotropic case.

The method has been applied to synthetic data. These data are generated by loading an elastic plate (of known rigidity) with two random fractal surfaces that represent the initial topographic and Moho loads on the plate, and modelling the resultant deflections through a finite-difference solution of the flexure partial differential equation. The post-loading topography and Bouguer anomaly are then analysed with the wavelet method to give a recovered rigidity, which agrees with the input rigidity to approximately 8% in the isotropic case. For anisotropy, the recovered weak directions agree with the model to less than 1 degree (Kirby and Swain, 2006; Swain and Kirby, 2006). The method has also been applied to real data over Australia, Europe, and North and South America (Tassara et al, 2007).

A significant contribution to gravity field recovery can be provided by the development of so-called topographic / isostatic gravity models, which are the result of forward gravity modelling mass density information on the Earth's topography and crust. Several such models have been constructed and analysed using various data sources and/or different istostatic compensation models (e.g. Tsoulis 2005, Tsoulis and Stray 2005a,b). Forward gravity modelling of crustal mass density information provided by the CRUST 2.0 global database has been performed by Tsoulis (2004a, b), Kuhn and Featherstone (2005) and Tsoulis et al. (2006). In Tsoulis (2004a, b) the gravitational effect of crustal mass anomalies have been analysed in the frequency domain using a spherical harmonic representation of Newton's integral. The same approach has been used in Kuhn and Featherstone (2005) in the development of a global forward gravity filed model (see section 4.5 below). Tsoulis et al. (2006) apply CRUST 2.0 for gravity field modelling over the Hellenic area and results in terms of isostatic gravity anomalies have been compared to two other independent method of determining the Moho discontinuity.

4.4 DTM creation and validation

Major contributions of SG members in the creation and validation of DTMs have been made. The development of a new global Digital Topographic Model (DTM2006.0), which incorporates the available SRTM data, as well as improved elevation data over Greenland and (parts of) Antarctica. DTM2006.0 was compiled in 30", 2', 5', 30', and 1° resolution. As with previous DTM compilations, this model provides information about terrain types, lake depths, and ice thickness (Pavlis et al. 2006a). A technique has been development and implemented that spectrally combines low degree gravity anomaly information, with high degree information implied by Residual Terrain Model (RTM) effects, to create "synthetic" gravity anomaly values over areas where gravity anomaly data are unavailable. These "synthetic" gravity anomaly values were necessary to develop gravitational model solutions complete to degree 2160, with proper spectral characteristics (Pavlis et al. 2006b).

A high resolution (1-arc-sec by 1-arc-sec, approx. 30 m by 30m) over Germany has been used to evaluate the currently released 3-arc-sec by 3-arc-sec SRTM terrain models (Denker 2005). Some considerable differences were found with a standard deviation of the differences of almost 8 m and maximum differences of up to 300 m. Further comparisons have been done with the global 30-arc-sec by 30-arc-sec GTOPO30.

4.5 Synthetic Earth gravity models: Derivation and validation

Two SEGMs have been developed at Curtin University of Technology since 2003 (Tsoulis and Kuhn 2006) of which one is a global model called CurtinSEGM (Kuhn and Featherstone 2003b, 2005) and one is a regional model called AusSEGM (Baran et al. 2006) defined over Australia only. The data for CurtinSEGM and AusSEGM are available via http://www.cage.curtin.edu.au/~kuhnm/CurtinSEGM/ and as electronic supplementary material from (Baran et al. 2006), respectively or from the corresponding authors of the models.

A synthetic [simulated] Earth gravity model (SEGM) generates exact and self-consistent gravity filed quantities and therefore is well suited to validate theories, algorithms and software used in gravity field modeling. A SEGM can be constructed using observations of the Earth's gravity field itself (source model), a reasonably realistic mass density distribution of the Earth's interior (source model) or a combination of both.

CurtinSEGM is a global source model SEGM using realistic parameters about the Earth's internal structure. The gravity field of CurtinSEGM is based on mass-density information of the topography, bathymetry, crust and mantle (Kuhn and Featherstone 2005), which have been forward modeled using a spherical harmonic representation of Newton's integral (e.g. Kuhn and Featherstone 2003a, Kuhn and Seitz 2005). The model is given by a spherical harmonic representation of the disturbing potential (up to and including degree an order 1440), which agrees reasonably well with empirical data such as given by EGM96.

AusSEGM is a regional SEGM over Australia using a combination of the source and effects model approach (Baran et al. 2006), which is specifically designed to validate regional gravimetric geoid determination theories, techniques and computer software. Currently AusSEGM is applied to test the geoid determination techniques used at Curtin University to construct the national geoid over Australia and at the University of New Brunswick to construct the geoid over Canada. AusSEGM provides synthetic gravity filed functionals (gravity, gravity anomaly and geoid height) on a regular 1-arc-min by 1-arc-min grid as well as arbitrary points with similar distribution as observed gravity stations. The long-wavelength effects part has been taken from an assumed errorless EGM96 (up to and including degree and order 360). The latter is a reasonable assumption in the context of the construction of a SEGM and ensures it replicates reasonably well the actual Earth's gravity field. A high-resolution (3-arc-sec. by 3-arc-sec) synthetic digital elevation model (SDEM), which is essentially a fractal surface based on the GLOBE v1 DEM has been constructed over Australia in order to model the short-wavelength source part of AusSEGM. Initial test have shown that AusSEGM is accurate to at least 30 μGal for gravity and gravity anomaly and 3 mm for the geoid height. Furthermore, a comparison of AusSEGM gravity values with 330,929 measured gravity values over Australia provided by Geoscience Australia (http://www.ga.gov.au/oracle/index.jsp) show a rather good agreement with most of the differences being less than 20 mGal (99.3 % of all values).

The results from a combination of geopotential model with regional terrestrial gravity data was investigated in Wolf and Denker (2005), Wolf (2006) and Wolf and Kieler (2006) with help of synthetic data including noise. Second order derivatives of the gravitational potential were computed, two methods (spectral combination with integral formulas and least-squares collocation) were applied (Wolf and Denker, 2005; Wolf, 2006). In the context of quasigeoid computation the integration using kernel modifications and different dimensioning of the integration area were investigated in (Wolf and Kieler, 2006). Noise was generated for the geopotential model as well as for the terrestrial gravity data in a correlated and uncorrelated version. The closed-loop results were confirmed by statistical error assessment.

4.6. Conference Contributions of SG Members

It is important to mention that apart from the contributions mentioned above many members of the SG were very active through presentations at several national and international conferences and workshops. However, these contributions are too numerous to mention them in detail here but titles are generally available through the group members own personal webpages.

4.7 Study Group Webpage

A webpage of the group's activities has been created, which summarises the activities of the SG as well as a list of relevant publications. Two mirrored versions of the web-page are located at Curtin University of Technology, Perth, Western Australia, as well as Aristotle University of Thessaloniki, Greece.

See html addresses:
http://www.cage.curtin.edu.au/~kuhnm/IAG_SG2.2/intex.html
http://users.auth.gr/~tsoulis/IAG_SG2.2/index.html

4.8 Meetings of the Study Group

During the period covered the SG had three official meetings of which the minutes are available from the SG's webpage.

1. Gravity, Geoid and Space Missions - GGSM2004. Porto, Portugal, August 30th to September 3rd, 2004.
2. Dynamic Planet 2005 Monitoring and Understanding a Dynamic Planet with Geodetic and Oceanographic Tools, Cairns, Australia, 22-26 August 2005
3. First International Symposium of the International Gravity Field Service (IGFS2006), August 28 - September 1, 2006, Istanbul, Turkey.

4.9 Conference Sessions

At the 1st International Symposium of the International Gravity Field Service (IGFS2006) the SG initiated session 8 on "*Terrain data and geopotential forward modelling*", which was convened by Michael Kuhn and Dimitrios Tsoulis. The session became an optimal platform for the group's members to present their current results, giving also the opportunity to other individuals to present their latest research in this field. The session attracted a total of 24 contributions (9 oral and 15 poster presentations). 11 authors responded positively to the call for submission of papers for inclusion to the Conference's Proceedings. All papers were led to a peer-reviewing process, after which 10 papers were accepted for publication. Our overall experience, judging from the quality and high standards of all presented papers, is that the SG's Session 8 has been receiver very well and was overall a success.

5. Future Work

The activity of the SG demonstrates that forward gravity modelling is a highly important topic in geodesy as well as other geo-sciences most notably geophysics. Obviously, this will not change in the foreseeable future. While the SG did not achieve all original aims manifested in its terms of references it contributed much to this topic warranting the recommendation of its continuation for another term of four years.

Some of the topics the SG could focus on during a second term could be:

- Evaluation of the high resolution SRTM terrain data in terms of forward gravity modelling. This includes the modelling or recovering of very high gravity signals or the detection of errors in the DTM.
- Global, regional and local gravity field recovery using forward gravity modelling results of the Earth's topography, crust and interior.

- Development and application of Synthetic Erath Gravity Models (SEGMs) based completely on forward gravity modelling. Applications could focus on the validation of techniques used to analyse data of the new geodetic satellite missions CHAMP, GRACE and in future GOCE, such as the derivation of mass changes from time varying gravity observations.
- Forward modelling of gravity inside the topographic masses.
- Overview/Summary/Collection of forward gravity modelling techniques including the comparison of forward gravity modelling software.

6. Publications

The following list of publication summarizes the ongoing activities of the SG members. It contains publications that were obtained by the chair up until the date of this report and demonstrates the increased activity of the members on the SG's core issues (publications of SG members are marked by an "*"). In addition the list contains some selected publications of individuals outside the group demonstrating the importance of the topic of forward modelling.

*Allasia G (2002): Approximating potential integrals by cardinal basis interpolants on multivariate scattered data. *Computers and Mathematics with Applications* 43(3-5): 275-287.

*Allasia G (2004): Recursive and parallel algorithms for approximating surface data on a family of lines or curves. In P. Ciarlini et al. (eds.): Advanced mathematical and computational tools in metrology VI, World Scientific, 2004, pp. 137-148.

*Allasia G, Besenghi R (2004): Approximation to surface data on parallel lines or curves by a near-interpolation operator with fixed or variable shape parameters. International Journal of Computational and Numerical Analysis and Applications, Vol. 5 No. 4 (2004), 317-337.

Benedek J (2004): The application of polyhedron volume element in the calculation of gravity related quantities. In: Meurers B (ed.), Proceedings of the 1st Workshop on International Gravity Field Research, Graz 2003, Special Issue of Österreichische Beiträge zu Meteorologie und Geophysik, Heft 31., pp. 99-106.

*Benedek J, Papp G (2006): Geophysical Inversion of On Board Satellite Gradiometer Data: A Feasibility Study in the ALPACA Region, Central Europe. In A. Kılıçoğlu R. Forsberg (eds.): Gravity Field of the Earth, Proceedings of the 1st International Symposium of the International Gravity Field Service (IGFS), 28 August – 1 September, 2006, Istanbul, Turkey (accepted).

*Baran I, Kuhn M, Claessens S, Featherstone WE, Holmes SA, Vaníček P (2006): A synthetic Earth gravity model specifically for testing regional gravimetric geoid determinations. Journal of Geodesy, DOI 10.1007/s00190-005-0002-z (with electronic supplement material).

Chakravarthi V, Raghuram HM, Singh SB (2002): 3-D forward gravity modelling of basement interfaces above which the density contrast varies continuously with depth. *Computers & Geosciences* 28: 53-57.

*Denker H (2005): Evaluation of SRTM3 and GTOPO30 terrain data in Germany. In: C. Jekeli, L. Bastos, J. Fernandes (eds.): Gravity, Geoid and Space Missions - GGSM2004, IAG

Internat. Symp., Porto, Portugal, 2004, IAG Symp., Vol. 129, 218-223, Springer Verlag, Berlin, Heidelberg, New York.

De Rossi A (2004): Spherical interpolation of large scattered data sets using zonal basis functions. Proceedings of the sixth International Conference on Mathematical Methods for Curves and Surfaces, July 1-6, 2004, Tromsø, Norway.

Farr TG, et al. (2007): The Shuttle Radar Topography Mission. Rev. Geophys., 45, RG2004, doi:10.1029/2005RG000183.

*Gitlein O, Denker H, Müller J (2005):. Local geoid determination by the spectral combination method. In: C. Jekeli, L. Bastos, J. Fernandes (eds.): Gravity, Geoid and Space Missions - GGSM2004, IAG Internat. Symp., Porto, Portugal, 2004, IAG Symp., Vol. 129, 179-184, Springer Verlag, Berlin, Heidelberg, New York.

*Kern M (2004): A comparison of data weighting methods for the combination of satellite and local gravity data. In: Sanso: V Hotine-Marussi Symposium on Mathematical Geodesy. Vol 127. pp 137-144.

*Kirby JF, Swain CJ (2004): Global and local isostatic coherence from the wavelet transform, *Geophysical Research Letters*, 31(24), L24608, doi: 10.1029/2004GL021569.

*Kirby JF Swain CJ (2006): Mapping the mechanical anisotropy of the lithosphere using a 2D wavelet coherence, and its application to Australia. *Physics of the Earth and Planetary Interiors*, Special Issue: Lithospheric Anisotropy, 158(2-4): 122-138.

*Kuhn M (2003): Geoid Determination with Density Hypotheses from Isostatic Models and Geological Information. *Journal of Geodesy*, 77: 50-65, DOI:10.1007/s00190-002-0297-y.

*Kuhn M, Featherstone WE (2003a): On the optimal spatial resolution of crustal mass distributions for forward gravity field modelling. In: Tziavos I.N. (ed) Gravity and Geoid 2002. 3rd Meeting of the International Gravity and Geoid Commission, Ziti Editions, Greece, 195-200.

*Kuhn M, Featherstone WE (2003b): On the construction of a synthetic Earth gravity model. In: Tziavos I.N. (ed) Gravity and Geoid 2002. 3rd Meeting of the International Gravity and Geoid Commission, Ziti Editions, Greece, 189-194.

*Kuhn M, Featherstone WE (2005): Construction of a synthetic Earth gravity model by forward gravity modelling. In F. Sansò (ed.): The Proceedings of the International Association of Geodesy: A Window on the Future of Geodesy, IAG Symposia 128:350-355, Springer Berlin, Heidelberg, New York.

*Kuhn M, Seitz K (2005): Evaluation of Newton's integral in space and frequency domain. In F. Sansò (ed.): The Proceedings of the International Association of Geodesy: A Window on the Future of Geodesy, IAG Symposia 128, Springer Berlin, Heidelberg, New York.

LaFer TR (1991): Standardization in gravity reduction. *Geophysics* 56(8): 1170-1178.

Li X, Chouteau M (1998): Three-dimensional gravity modelling in all space. *Surveys in Geophysics* 19: 339-368.

Mooney WD, Laske G, Masters TG (1998): CRUST 5.1: A global crustal model at 5° x 5°, *J Geophys Res*, 103:727-747.

*Novák P, Bruton AM, Bayoud FA, Kern M, Schwarz KP (2003): On numerical and data requirements for topographical reduction of airborne gravity in geoid determination and resource exploration. *Bollettino di Geodesia e Scienze Affini* 62: 103-124.

⋆Novák P, Grafarend EW (2005): The ellipsoidal representaiton of the topographical potential and its vertical gradient, *J Geodesy* 78: 691-706.

⋆Novák P, Grafarend EW (2006): The effect of topographical and atmospheric masses on spaceborne gravimetric and gradiometric data, *Studia Geophysica et Geodaetica* 50: 549-582.

⋆Novák P, Kern M, Schwarz K-P, Heck B (2003): Evaluation of band-limited topographical effects in airborne gravimetry. *J Geodesy* 76: 597-604.

⋆Novák P (2006): Evaluation of local gravity field parameters from high resolution gravity and elevation data. *Contributions to Geophysics and Geodesy* 36: 1-33.

⋆Novák P (2007): Gravity reduction using a general method of Helmert's condensation. *Acta Geodaetica et Geophysica Hungarica* 42: 83-105.

Novell DAG (1999): Gravity terrain corrections – an overview. *J Applied Geophysics* 42: 117-134.

Parker RL (1995): Improved Fourier terrain correction, Part I. *Geophysics* 60(4): 1007-1017.

Parker RL (1996): Improved Fourier terrain correction, Part II. *Geophysics* 61(2): 365-372.

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